Part I

Assignment 4

Exercise 1

We say a set $F \subset \mathbb{R}$ is *closed* if its complement $F^c := \mathbb{R} \backslash F$ is open. Since $\emptyset$ and $\mathbb{R}$ are open, it follows that $\emptyset$ and $\mathbb{R}$ are closed as well.

(a) Let $a, b \in \mathbb{R}$ with $a < b$. Prove that $[a, b]$ is closed.

(b) Is the set $\mathbb{Z} \subset \mathbb{R}$ closed? Provide a proof to substantiate your claim.

(c) Is the set of rationals $\mathbb{Q} \subset \mathbb{R}$ closed? Provide a proof to substantiate your claim.