MIT OCW Real Analysis

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Week 1

Exercise 0.3.6

a) Wanting to show: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

In order to prove this equivalence, we have to prove the implication both ways. We use two lemmas for this.

Lemma 1.1 — $A \cap (B \cup C) \implies (A \cap B) \cup (A \cap C)$

Let $x \in A \cap (B \cup C)$. By the definition of set intersection, $x \in A$ and $x \in B \cup C$. By the definition of set union, $x \in A$ and ($x \in B$ or $x \in C$). From propositional logic we know that for propositions P, Q and R the following holds: $P \wedge (Q \vee R) \iff (P \wedge Q) \vee (P \wedge R)$. So, substituting for this particular case yields ($x \in A$ and $x \in B$) or ($x \in A$ and $x \in C$). Using the definition of set intersection again gets $x \in A \cap B$ or $x \in A \cap C$. Using the definition of set union again gives $x \in (A \cap B) \cup (A \cap C)$. □

Lemma 1.2 — $(A \cap B) \cup (A \cap C) \implies A \cap (B \cup C)$

Let $x \in (A \cap B) \cup (A \cap C)$. By the definition of set union, $x \in (A \cap B)$ or $x \in (A \cap C)$. By the definition of set intersection, ($x \in A$ or $x \in B$) and ($x \in A$ or $x \in B$). Using the same propositional logical equivalence as in Lemma 1.1, this gives $x \in A$ and ($x \in B$ or $x \in C$). Wrapping up, we use the definition of set union to get $x \in A$ and $x \in B \cup C$ and the definition of intersection to get $x \in A \cap (B \cup C)$. □

Using Lemma 1.1 and 1.2, we get the desired equivalence of $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. □

b) Wanting to show: $A \cup (B \cap C) = (A \cup B) \cap (a \cup C)$

This proof is so similar to a) that it feels like a waste of time and will therefore be left to the reader.